Market Efficiency and Long-range Dependence: Evidence from the Tehran Stock Market

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ABSTRACT

In an efficient market, the price process must follow a random walk, and price changes must be random. The presence of short and long-range dependence in the stock price process rejects the random walk and resulting in market inefficiency. The main objective of this paper is to examine the Tehran exchange market inefficiency attributable to the presence of long-range dependence in the market. To do so, we study the time-varying long-range dependence in the Tehran Stock Exchange log-return process using financial econometrics models. We provide clear statistical evidence that the mean log-return price process of the Tehran exchange market is a non-stationary process with short range memory. Our finding indicates that shocks in the volatility of the Tehran stock market decay more slowly than an exponential decay. The results provide strong evidence in rejecting the random walk and the market efficiency hypotheses in the Tehran stock exchange market.

Keywords: Long range dependence; market efficiency; Tehran stock exchange, financial econometrics.

1. INTRODUCTION

In an efficient market, the price process follows a random walk, a term that indicates stock price changes must be random, and stock price discovery should be instantaneous [1]. The notion of long-range dependence (LRD) is associated with a time series whose autocovariance function decreases gradually like a power function as the lag between two observations rises. The time series with LRD has been analyzed widely and used in various fields, including economics, finance, electrical engineering, and elsewhere. The presence of LRD in stock price processes rejects the random walk hypothesis and the market efficiency hypothesis.

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Mandelbrot [2] was among the first that showed the presence of long-dependence memory in the stock market. After that, LRD became one of the fascinating topics in economics and finance. Fama and French [3] examined the auto-correlations of stock returns for increasing holding periods. Their result indicated that there was a slowly mean-reverting component of stock prices expected returns. Lo and MacKinlay [4] study the hypothesis of random walk for stock price using a simple specification test based on variance estimators. Their result indicated that the stock price process does not follow the random walk process. Jacobsen [5] studied whether the price process is mean-reverting, using data from the United States and 17 other countries. They showed that there is a negative auto-correlation over longer horizons in the stock market, demonstrating non-random-walk price behavior.

Admitting the fact that stock market returns exhibit LRD, many authors have investigated the presence of LRD both in the mean and volatility of the stock return process. Willinger et al. [6] examined the LRD in the mean of daily stock returns using a modified adjusted range. They found empirical evidence of long-range dependence with a very low degree in the stock return process. Using 34 stock index returns, Christodoulou-Volos and Siokis [7] showed the most stock return are long-range dependent. The test p-value used in their study provided strong evidence of LRD in either stock or exchange rate returns.

Cajueiro and Tabak [8] extended the methodology used for testing LRD in the mean of stock returns. They implemented a new bootstrap methodology to asses LRD in 41 countries in Europe, Asia, Latin America, and the Middle East. They showed that developed economies are weak-form efficient, while Latin American economies are more inefficient than Asian economies. Aye et al. [9] examined the existence of LRD in the stock market returns for BRICS countries. They applied the fractionally integrated generalized autoregressive moving average (FIARMA) model. Their study’s results indicated strong evidence supporting the existence of long memory in daily stock returns for the BRICS countries.

Despite the extensive research into the presence of LRD in the mean of stock return, usually, in US markets, some researchers investigated the presence of LRD in the variance of stock return. Crato and Lima [10] examined the presence of LRD in the conditional variance of stock return. The empirical result using semi-parametric and non-parametric statistical methods indicated the presence of LRD in the squared returns series. Ray and Tsay [11] investigated the presence of LRD in the volatility of the stock market for the daily return of the S&P 500 index using fractionally integrated processes. Their results indicated the strength of LRD in the stock volatilities. The presence of LRD in the volatility of the Brazilian Stock Market was examined by Cavalcante [12] by applying re-scaled variance V/S statistic and fractionally integrated autoregressive conditional heteroscedasticity (FIGARCH) model.

In their paper, Cheong and Pei [13] used the adjusted range (R/S) statistic to assess the time-varying LRD behaviors of the S&P500 volatility index. Shirvani [14] applied the FIGARCH model to obtain a time-varying LRD index for the mean and variance of the S&P 500 log-returns during the 2008 market crash. The results of his study indicate shock in market decay at an exponential rate in the mean model and more slowly than an exponential decline in the volatility of the S&P 500 returns during the 2008 market crash.

Despite the extensive research about European and US market indices, there is some research available regarding the presence of LRD in Asian indices. Jacobsen [15] used the modified rescaled range statistic to study LRD behavior in Japan stock market. They found evidence of LRD for the Japan index. Cajueiro and Tabak [16] examined the presence of long-range dependence in China, Hong Kong, and Singapore stock market indices by using Hurst’s exponent measure. They found that Hong Kong is the most efficient market, followed by Chinese and Singapore. The results of Mishra et al. [17] results indicate the presence of LRD in returns series for six Indian stock market indices.

Therefore, the main objective of this paper is to examine the time-varying long-range dependence in the Tehran Stock Exchange (TSE) using rolling estimation methods. We consider the presence of LRD in the mean and volatility of the TSE log-returns process.

The FIARMA model is employed to capture LRD in the mean log-return process. To test the

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1 BRIC is an acronym for Brazil, Russia, India, and China, a group of countries considered to be developing countries.
presence of LRD in log-returns fluctuations, we apply the FIGARCH model.

In general, we employ the standard FIARMA and FIGARCH models, ARFIMA(1,d,1) - GARCH(1,1) and ARMA(1,1) - FIGARCH(1,d,1) [18,19]. Although Shirvani and Volchenkov [20] confirmed the presence of short memory dependence in the TSE log-returns, we emphasize the time-varying values for d to see investors trading partners during 2009-2018. The results indicate the TSE log-return process is neither stationary nor a mean-reverting process. The shocks in the Tehran stock market decay more slowly than an exponential decay in the mean model. The estimated values for d in ARMA(1,1)-FIGARCH(1,d,1) model are varying in (0.0,1.0) indicating a lack of power to distinguish between a truly short and long memory behavior in TSE volatility log-return process.

This paper is organized as follows. The next section describes the data source and data validation. In Section 3, we review the methodology used for modeling the mean and volatility of the TSE log-returns process. We use econometrics models to examine the presence of LRD in the mean of the TSE log-return process in Section 4. In Section 5, we obtain a time-varying index for LRD to consider the dependence behavior in the log-return volatility of the TSE index. Section 6 summarizes our findings.

2. DATA SOURCE, DATA VALIDATION AND PREPARATION

We use daily adjusted closing prices for the TSE index collected from the official web site of the Tehran Stock Exchange from 28/02/2005 to 14/11/2018. 2 The TSE price’s timestamps transformed from Persian to Western dates. Fig. 1 shows the TSE daily price process for the period from 28/02/2005 to 14/11/2018. As we observe, the volatility of the TSE index was very high in 2014-2015. It is worth to note that Fig. 1 provides empirical evidence of rejecting normal assumption and supporting a heavy tail distribution for the TSE market log-return process, especially during 2014-2015.

3. METHODS

In this section, we shortly explain the methodology that we are applying in this research. First, we carry out the traditional unit-root non-stationary tests to examine whether the price process is non-stationary. The p-value (~0.86) of the Augmented Dickey-Fuller test provides strong evidence of unit-root non-stationary process. As in our analysis, we need to work with a stationary process; we obtain the log-returns of the risky asset by the logarithmic return as follows

$$r(t) = \ln \frac{S(t)}{S(t-1)},$$

where $S(t)$ is the price of S&P 500 at day $t$.

Fig. 2 shows the TSE log-return time-series process from 28/02/2005 to 14/11/2018. Again, we can recognize the volatility clustering and the heavy tail assumption from Fig. 2.

Baillie [21] defined long memory as the dependence structure of a time series that represents temporal dependence between observations distant, discrete, and far apart in time. In the financial time series, if shocks to the market tend to decay at a hyperbolic rate, it refers to a long-range memory process, whereas in the case of a short-range dependence process and the shocks to the market decay at an exponential rate. The autocorrelation function $\rho_j$ in LRD process at lag $j$ is define

$$\lim_{n \to \infty} \sum_{j=-n}^{n} |\rho_j| = \infty,$$

where $n$ is the number of observations.

Econometrics is an active field that widely used in economics, business, finance, statistics, probability, and applied mathematics [22,23, 24,20,25,26,27]. Financial econometrics is a mixture of economics, business, finance, and statistics. It provides quantitative methods to explain quantitative problems in finance and marketing. Over the last two decades, an enormous number of new financial product has been created by applying financial econometrics models. Trindade et al. [28] constructed an index to measure of the US citizenry and applied econometric models to price the index.

Here, we are going to create two-time varying indices to measure the LRD in the mean and volatility of the TSE market return process. We apply the Autoregressive Moving Average (ARMA)-Autoregressive Conditional Heteroscedasticity (GARCH) model [29]. The ARMA(1,1)-GARCH(1,1) is the standard tool for modeling the mean and volatility of stock returns. It is defined as

\[ r(t) = \ln \frac{S(t)}{S(t-1)}, \]
\[
\begin{aligned}
  r_t &= \mu + \phi(r_{t-1} - \mu) + \theta a_{t-1} + a_t \\
  a_t &= \varepsilon_t \sigma_t, \quad \varepsilon_t \sim iid, \\
  \sigma_t^2 &= \gamma + \alpha a_{t-1}^2 + \beta \sigma_{t-1}^2
\end{aligned}
\]  

(3)

\(\mu, \sigma, \alpha, \beta, \gamma, \delta, \phi, \theta\) are the model parameters.

\(\varepsilon_t\) is a white noise process, \(a_t\) is referred to the market shocks, \(\alpha \geq 0, \beta \geq 0, \gamma \geq 0, \delta, \phi, \theta\) are the model parameters.

Granger and Joyeux [30], Hosking [31], and McLeod and Hipel [32] were among the first statisticians and economists that used fractional autoregressive integrated moving-average (FARIMA) time series to study Long-memory time series. Tsay (2005) defined LRD in term of FARIMA time series as follows

\[
\phi(L)(1 - L)^d r_t = \theta(L)a_t; 0 \leq d \leq 1,
\]

(4)

where \(d\) is the fractional integration parameter, \(L\) is the lag operator, and \(a_t\) refers to market shocks. When \(d > 0.5\), \(r_t\) is a nonstationary long memory process. When \(d < 0.5\), \(r_t\) is anti-persistent time series.

Fig. 1. TSE price process from 28/02/2005 to 14/11/2018

Fig. 2. TSE log-return process from 28/02/2005 to 14/11/2018
The time-varying volatility model broadly has been applied in economic, finance, agriculture, business, physical science, and engineering. Engle [19] and Bollerslev and Mikkelsen [33] proposed a new class of fractionally integrated GARCH (FIGARCH) model for characterizing financial market volatility. In the FIGARCH model, the time series volatility of the market is defined as

\[ (1 - \alpha(L))(1 - L)^\delta \sigma_t^2 = \gamma_0 + (1 - \beta(L))a_t^2. \]  

Equation 5

4. LONG RANGE DEPENDENCE IN TSE LOG-RETURN PROCESS

In this section, we use the standard frictional time series model FIARMA(1,d,1)-GARCH(1,1) to examine the presence of LRD in the TSE mean log-return process. The standard FIARMA(1,d,1)-GARCH(1,1) (see Tsay, 2005) is defined as

\[
\begin{align*}
(1 - \phi L)(1 - L)^\delta(r_t - \mu) &= \theta a_{t-1} + a_t, & 0 \leq d \leq 1, \\
\alpha_t &= \epsilon_t + \epsilon_t - \text{iid,} \\
\sigma_t^2 &= \gamma + \alpha_t^2 + \beta \sigma_{t-1}^2.
\end{align*}
\]

Equation 6

where \( \epsilon_t \) are iid with Student-t distribution to have a reasonable fit to the data set.

To estimate the model parameter, we apply rolling estimation methods with the rolling windows of 1000 observations (4 years data set). We fit the FIARMA(1,d,1)-GARCH(1,1) model to the first 1000 observations and estimate parameters. Moving the rolling window by one observation, we again fit the FIARMA(1,d,1)-GARCH(1,1) model to the second 1000 observation and estimate the model parameters. We repeat this methodology for \( n = 1, 2, ..., 2203 \). The exact maximum likelihood is used for estimating \( d \) in our model. Hurst–Mandelbrot statistic Terdik [34] is used to test the significance of the estimated value for \( d \) in 5% significance level. The model parameters are estimated by implementing the rugarch package in R [35].

Fig. 3 exhibits time-varying LRD in the mean of the TSE log-return process. As we observe, the estimated values for \( d \) are varying in \((0.05, 0.48)\), indicating intermediate memory (ant persistence). This result suggests that the TSE log-return process is neither stationary nor mean-reverting. The shocks in the Tehran stock market decay more slowly than an exponential decay in the mean model. Thus, the price may be predictable at short horizons.

Fig. 4 shows the time-varying p-values for the Hurst–Mandelbrot test. The null hypothesis of the test is \( d = 0 \) corresponding to stationary and invertible ARMA model. In 95% of the time, the p-values are less than 0.05 indicating strong evidence of the TSE return non-stationary process. These results indicate that the TSE log-return process is affected by the shocks to the market and trading partners during the sample period and caused a significant non-stationary process.

Fig. 3. Time-varying LRD (d) in the TSE mean log-return process
5. LONG RANGE DEPENDENCE IN TSE VOLTILITY

The ARMA(1,1)-FIGARCH(1,d,1) is applied to examine the presence of LRD in the volatility of the TSE log-return process. In the ARMA(1,1)-FIGARCH(1,d,1) model, fractional integration refers to long-range dependence in the volatility of the time series process indicating a non-stationary timeseries. A FIGARCH time-series process has infinite memory as all past values of the market shocks are embedded in the volatility model. The standard ARMA(1,1)-FIGARCH(1,d,1) is defined as

\[
\begin{align*}
\gamma_t &= \mu + \phi(\gamma_{t-1} - \mu) + \theta \sigma_{t-1} + \epsilon_t, \\
\sigma_t &= \epsilon_t \sigma_t, \quad \epsilon_t \sim i.i.d, \\
(1 - \alpha L)(1 - L)^d \sigma_t^2 &= \gamma + (1 - \beta L)\sigma_t^2,
\end{align*}
\]  

Here, we assume Student-t distribution for model innovation ($\epsilon_t$) as we observed the volatility clustering in the log-return time-series process in Fig. 2.

We use rolling estimation methods to estimate ARMA(1,1)-FIGARCH(1,d,1) model parameters. The rolling window of 1000 observations is considered in the estimation process. The rolling
estimation is repeated for \( n = 2203 \) times to have a stable time series for \( d \) in \( FIGARCH(1,d,1) \) model. Again, the maximum likelihood estimation method is used, and the significance of \( d \) is tested using Hurst–Mandelbrot statistics.

Fig. 5 exhibits time-varying LRD in the TSE volatility log-return process. As we observe, the estimated values for \( d \) are varying in \((0.0,1.0)\), indicating short, long memory, and non-stationary process. In Fig. 5, the time-varying LRD index is divided into two periods: before and after 2015. The non-stable and time-varying values of \( d \) indicate the lack of power to distinguish between a concise and long memory behavior in the TSE volatility log-return process. The presence of a structural break is observed in the dependence behavior in the TSE volatility log-return process. After 2015, the log-return volatility of the TSE process is genuinely a unit root (non-stationary) process. It is another indication that the log-return process does not follow a random walk. In other words, the volatility of the stock return can be discovered by using historical volatility. Thus, according to the results, we can reject the random walk and the market efficiency hypotheses in the TSE market.

The time-varying \( p \)-values of the null hypothesis for the presence of LRD in the volatility of the TSE log-return process are plotted in Fig. 6. The rejection rate of the null hypothesis (having a stationary process) is 97%. These results indicate robust evidence supporting the existence of dependence in daily stock log-returns of the Tehran stock market. Thus, the inefficiency of the TSE market is resulted.

6. DISCUSSION AND CONCLUSION

We studied the time-varying long-range dependence in the Tehran Stock Exchange (TSE) using financial econometrics models. The rolling estimation methods were applied to estimate model parameters. We consider the presence of LRD in the mean and volatility of the TSE log-returns process by applying fractional integrated ARMA and GARCH models, respectively. The result of the fractional ARMA model indicates that the TSE log-return process is neither stationary nor mean-reverting. The shocks in the Tehran stock market decay more slowly than an exponential decay in the mean model. The non-stable and time-varying values of the fractional FARCH model indicate the lack of power to distinguish between a concise and long memory behavior in the TSE volatility log-return process. According to the results of both models, we reject the random walk and the market efficiency hypotheses in the TSE market.

COMPETING INTERESTS

Authors have declared that no competing interests exist.
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